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NATIONAL SUPERCOMPUTER CENTER IN GUANGZHOU

Compilation Principle 编译原理

第7讲：语法分析(4)

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Quiz Questions

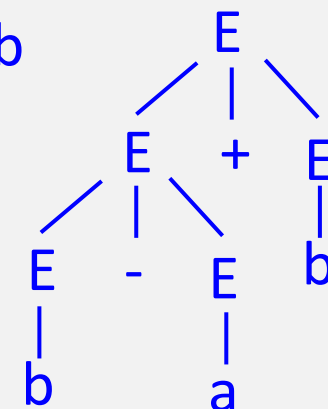


- Q1: for grammar $E \rightarrow E-E \mid E+E \mid a \mid b$, and input $b-a+b$, give one rightmost derivation.

$E \rightarrow E - E \rightarrow E - E + E \rightarrow E - E + b \rightarrow E - a + b \rightarrow b - a + b$

$E \rightarrow E + E \rightarrow E + b \rightarrow E - E + b \rightarrow E - a + b \rightarrow b - a + b$

- Q2: plot parse tree of the derivation in Q1.



- Q3: briefly describe top-down parsing.

Mimics leftmost derivation, expand the start symbol to input string.

- Q4: why top-down parsing cannot handle left recursive grammars?

Repeatedly expanding without consuming any input symbol.

- Q5: is grammar $S \rightarrow T a \mid a, T \rightarrow S$ left recursive? Why?

YES. $S \rightarrow T a \rightarrow S a$ (indirect left-recursive).

Predictive Parsers[预测分析]

- In recursive descent with backtracking[有回溯]:
 - At each step, many choices of production to use
 - Backtracking used to undo bad choices
- A parser with **no backtracking**[无回溯]: **predict** correct next production given next input terminal(s)? [以下面一些输入来预测]
 - If first terminal of every alternative production is **unique**, then parsing requires no backtracking[候选产生式开始符号唯一]
 - If not unique, grammar cannot use predictive parsers[不唯一]

$A \rightarrow aBD \mid bBB$

$B \rightarrow c \mid bce$

$D \rightarrow d$


parsing input “**abcd**” requires no backtracking

?: 如果只往前看一个，那么next terminal其实就是current terminal，即要匹配的那个（注意backtrack是完全不看）

Predictive Parsers (cont.)

- A predictive parser chooses the production to apply solely on the basis of[选取产生式的依据]
 - Next input symbol(s)[下一输入符号/终结符]
 - Current nonterminal being processed[当前正处理的非终结符]
 - Patterns in grammars that prevent predictive parsing[并非总是能预测分析]
 - **Common prefix**[共同前缀]:
 $A \rightarrow \alpha\beta \mid \alpha\gamma$
Given input terminal(s) α , cannot choose between two rules
 - **Left recursion**[左递归]:
 $A \rightarrow A\beta \mid \alpha$
Lookahead symbol changes only when a terminal is matched
- What is the language of the grammar? $\alpha\beta^*$

Rewrite Grammars for Prediction[改写]

- **Left factoring**[左公因子提取]: removes common left prefix
 - In previous example: $A \rightarrow \alpha\beta \mid \alpha\gamma$
 - can be changed to $stmt \rightarrow \text{if expr then stmt else stmt} \mid \text{if expr then stmt}$
 $A \rightarrow \alpha A'$  $stmt \rightarrow \text{if expr then stmt } S'$
 $A' \rightarrow \beta \mid \gamma$ $S' \rightarrow \text{else stmt} \mid \epsilon$
 - After processing α , A' can choose between β or γ
(assuming β or γ do not start with α) 推迟选择, 直到可区分
- **Left-recursion removal**[左递归消除]: same as recursive descent
 - In previous example: $A \rightarrow A\beta \mid \alpha$
 - can be changed to
 $A \rightarrow \alpha A'$
 $A' \rightarrow \beta A' \mid \epsilon$
 - After processing α , A' can choose between β or ϵ
(assuming β doesn't start with α or A' isn't followed by α)

LL(k) Parser / Grammar / Language

- **LL(k) Parser**

- A predictive parser that uses k lookahead tokens
- **L**: scans the input from **left to right**[从左往右]
- **L**: produces a **leftmost derivation**[生成最左推导]
- **k**: using k input symbols of lookahead at each step to decide[向前看k个符号]

- **LL(k) Grammar**

- A grammar that can be parsed using an LL(k) parser
- $LL(k) \subset CFG$
 - Some CFGs are not LL(k): common prefix or left-recursion

- **LL(k) Language**

- A language that can be expressed as an LL(k) grammar

- Many languages are LL(k) ...

- In fact many are **LL(1)**!

LL(k) Parser Implementation[实现]

- Implemented in a recursive or non-recursive fashion[递归/非递归]
 - Recursive: recursive descent (recursive function calls, implicit stack)
 - Non-recursive: explicit stack to keep track of recursion[栈]
- Recursive LL(1) parser for: $A \rightarrow B \mid C, B \rightarrow b, C \rightarrow c$
 - Parser consists of small functions, one for each non-terminal

```
void A() {  
    token = peekNext(); // lookahead token  
    switch(token) {  
        case 'b': // 'B' starts with 'b'  
            B(); // call procedure B()  
        case 'c': // 'C' starts with 'c'  
            C(); // call procedure C()  
        default: // Reject  
            return;  
    }  
}
```

LL(k) Parser Implementation (cont.)

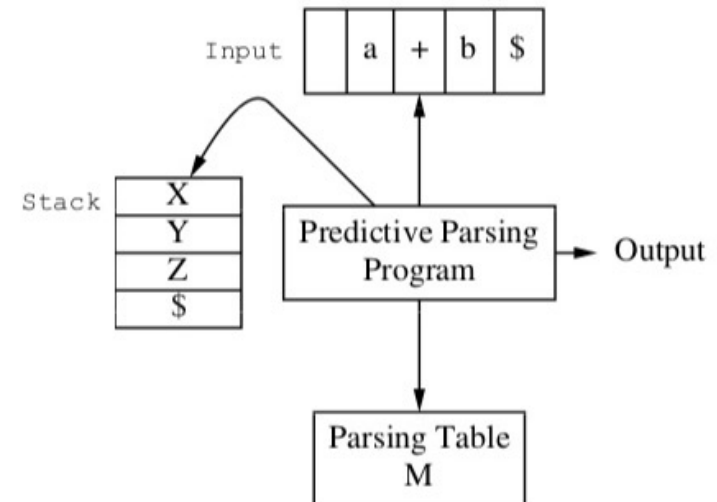
- Recursive LL(1) parser for: $A \rightarrow B \mid C, B \rightarrow b, C \rightarrow c$

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        case 'b': // 'B' starts with 'b'  
            B(); // call procedure B()  
        case 'c': // 'C' starts with 'c'  
            C(); // call procedure C()  
        default: // Reject  
            return;  
    }  
}
```

- Is there a way to express above code more concisely?[简洁]
 - Non-recursive LL(k) parsers use a **state transition table** (just like finite automata)[状态转换表]
 - Easier to automatically generate a non-recursive parser[自动化]

LL(1) Parser[非递归]

- Table-driven parser[表驱动]: amenable to automatic code generation (just like lexers)
 - **Input buffer**: contains the string to be parsed, followed by \$
 - **Stack**: holds unmatched portion of derivation string, \$ marks the stack end
 - **Parse table** $M[A, b]$: an entry containing rule “ $A \rightarrow \dots$ ” or error
 - **Parser driver** (a.k.a., predictive parsing program): next action based on <stack top, current token>
 - ▣ **Reject** on reaching error state
 - ▣ **Accept** on end of input & empty stack



A stack records frontier of parse tree

- Non-terminals that have yet to be expanded
- Terminals that have yet to be matched against the input
- Top of stack = leftmost pending terminal or non-terminal

?: The current token is treated as lookahead token.

LL(1) Parse Table: Example

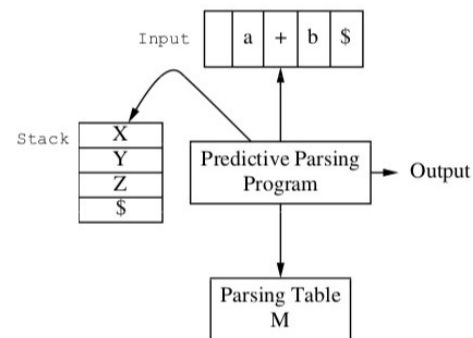
table	int	*	+	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'			$E' \rightarrow +E$		$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow \text{int } T'$			$T \rightarrow (E)$		
T'		$T' \rightarrow *T$	$T' \rightarrow \epsilon$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$

$E \rightarrow TE'$
 $E' \rightarrow +E \mid \epsilon$
 $T \rightarrow \text{int}T' \mid (E)$
 $T' \rightarrow *T \mid \epsilon$

- Implementation with 2D parse table
 - **First column** lists all non-terminals in the grammar
 - I.e., leftmost non-terminal in derivation
 - **First row** lists all possible terminals in the grammar and \$
 - I.e., next input token
 - A **table entry** contains one production
 - One action for each <non-terminal, input> combination
 - It “predicts” the correct action based on one lookahead
 - No backtracking required

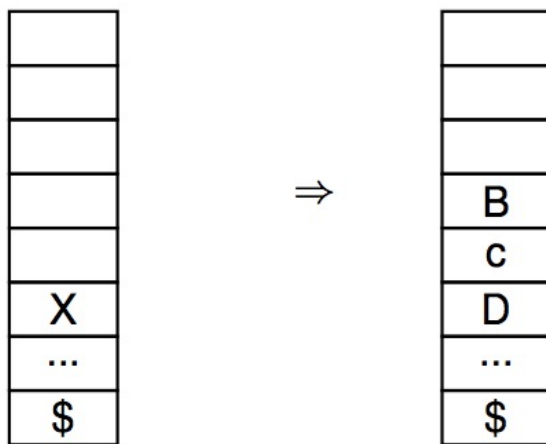
LL(1) Parsing Algorithm[算法]

- Initial state[初始态]
 - **Input** tape: input tokens followed by '\$'
 - **Stack**: start symbol followed by '\$' at bottom
- General idea[总体思路]: repeat one of two actions
 - **Expand** symbol at top of stack by applying a production
 - **Match** terminal symbol at top of stack with input token
- Step-by-step[每步操作] parsing based on $\langle X, a \rangle$
 - X: symbol at the top of the stack
 - a: current input token
 - If $X \in T$, then
 - If $X == a == \$$, parser halts with “success”
 - If $X == a \neq \$$, successful match, pop X from stack and advance input head
 - If $X \neq a$, parser halts and input is **rejected**
 - If $X \in N$, then
 - If $M[X, a] == 'X \rightarrow \text{RHS}'$, pop X and push RHS to stack
 - If $M[X, a] == \text{empty}$, parser halts and input is **rejected**



Push RHS in Reverse Order[逆序入栈]

- For $\langle X, a \rangle$
 - X : symbol at the top of the stack
 - a : current input token
- If $M[X, a] = "X \rightarrow BcD"$



- Performs the leftmost derivation: $\alpha X \beta \Rightarrow \alpha BcD \beta$
 - α : string that has already been matched with input
 - β : string yet to be matched, corresponding to the ... above

Apply LL(1) Parsing to Grammar[应用]

- Consider the grammar

$$E \rightarrow T+E | T$$

$$T \rightarrow \text{int} * T | \text{int} | (E)$$

– Left recursion? **NO!**

– Left factoring? **YES.** $E \rightarrow T+E | T$, $T \rightarrow \text{int} * T | \text{int}$

- After rewriting grammar, we have

$$E \rightarrow TE'$$

$$E' \rightarrow +E | \varepsilon$$

$$T \rightarrow \text{int}T' | (E)$$

$$T' \rightarrow *T | \varepsilon$$

Use the Parse Table

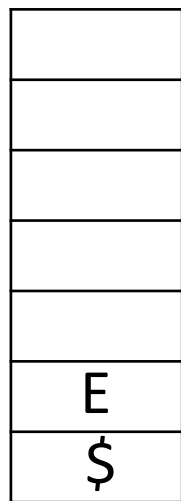
- To recognize "int * int"

$$E \rightarrow TE'$$

$$E' \rightarrow +E \mid \epsilon$$

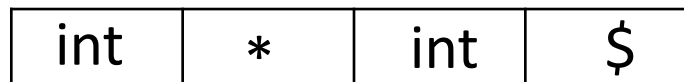
$$T \rightarrow intT' \mid (E)$$

$$T' \rightarrow *T \mid \epsilon$$



stack

input



parser driver

table	int	*	+	()	\$
E	E → TE'			E → TE'		
E'			E' → +E		E' → ε	E' → ε
T	T → int T'			T → (E)		
T'		T' → *T	T' → ε		T' → ε	T' → ε

Use the Parse Table

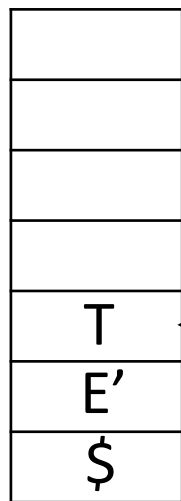
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stack

input

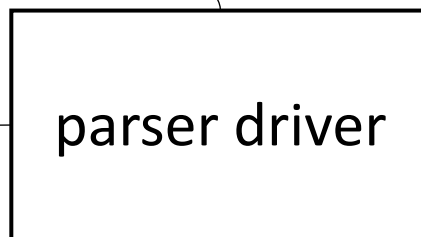


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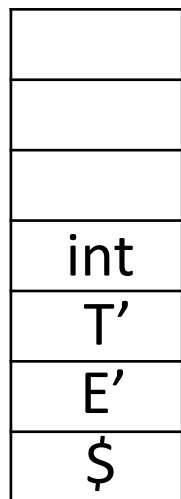
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stack

input

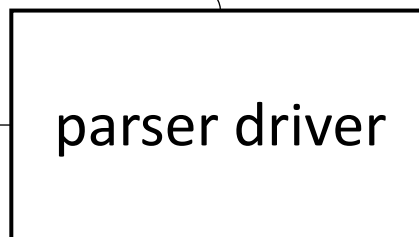


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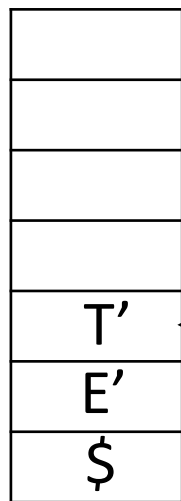
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stack

input

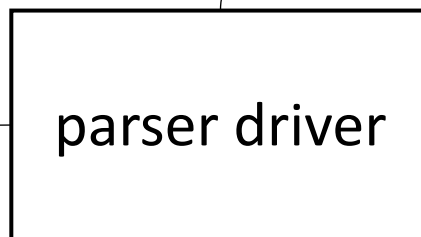


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T'		$T' \rightarrow *T$	$T' \rightarrow \epsilon$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$

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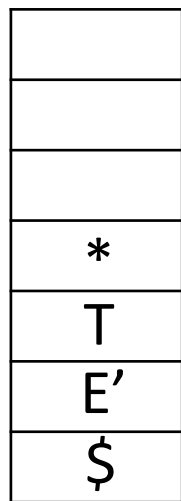
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stack

input

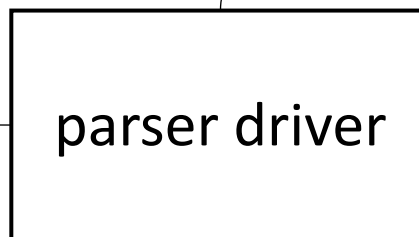


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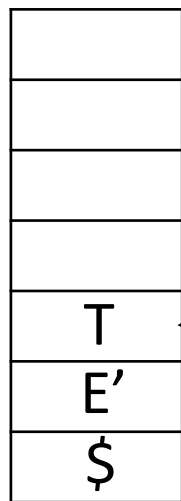
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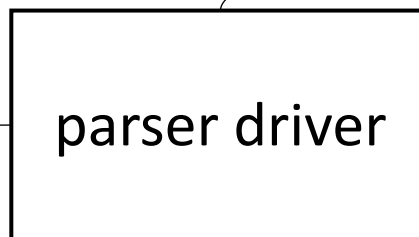


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Use the Parse Table

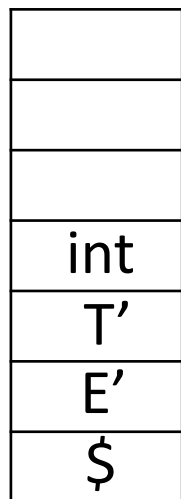
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stack

input

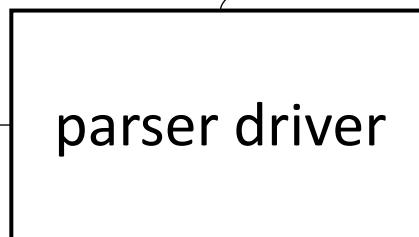


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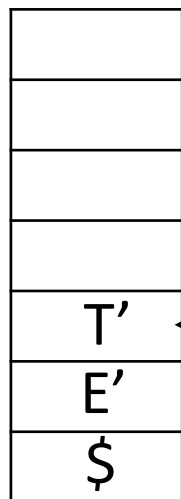
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stack

input

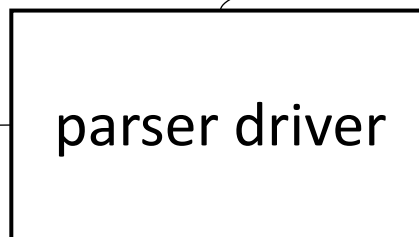


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T'		$T' \rightarrow *T$	$T' \rightarrow \epsilon$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$

Use the Parse Table

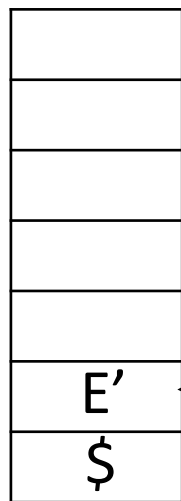
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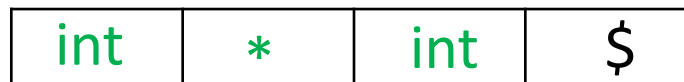
$$T \rightarrow intT' \mid (E)$$

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stack

input



parser driver

table	int	*	+	()	\$
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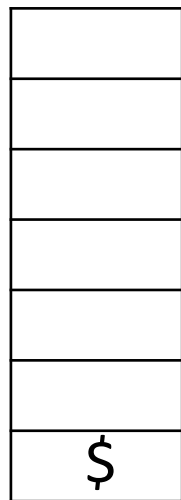
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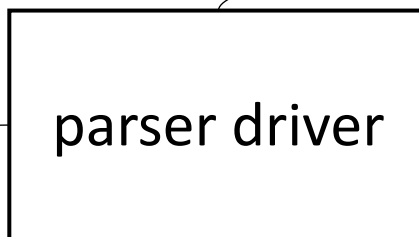
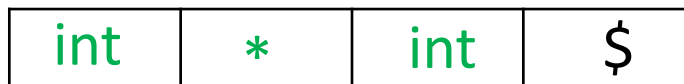
$$T \rightarrow intT' \mid (E)$$

$$T' \rightarrow *T \mid \epsilon$$



stack

input



ACCEPT!

table	int	*	+	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'			$E' \rightarrow +E$		$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow int T'$			$T \rightarrow (E)$		
T'		$T' \rightarrow *T$	$T' \rightarrow \epsilon$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$

Recognize Sequence[解析过程]

Matched	Stack	Input	Action
	E \$	int * int \$	$E \rightarrow TE'$
	T E' \$	int * int \$	$T \rightarrow int T'$
int	int T' E' \$	int * int \$	match
int	T' E' \$	* int \$	$T' \rightarrow *T$
int	* T E' \$	* int \$	match
int *	T E' \$	int \$	$T \rightarrow int T'$
int *	int T' E' \$	int \$	match
int * int	T' E' \$	\$	$T' \rightarrow \epsilon$
int * int	E' \$	\$	$E' \rightarrow \epsilon$
int * int	\$	\$	Halt and accept

$E \rightarrow TE'$

$E' \rightarrow +E \mid \epsilon$

$T \rightarrow intT' \mid (E)$

$T' \rightarrow *T \mid \epsilon$

Input: int * int

- 'Matched + Stack' constructs the sentential form[句型]
- Actions correspond to productions in leftmost derivation