



中山大學
SUN YAT-SEN UNIVERSITY



国家超级计算广州中心
NATIONAL SUPERCOMPUTER CENTER IN GUANGZHOU

Compilation Principle 编译原理

第9讲：语法分析(6)

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Review Questions

- Q1: how to build the LL(1) parse table?

Two sets: FIRST, FOLLOW

- Q2: for the grammar, what is FIRST(A) and FOLLOW(B)?
FIRST(A) = {b, d, ϵ }, FOLLOW(B) = {d, a}
- Q3: which one is typically used, LL(0), LL(1), LL(2) ...? Why not others?

LL(1). LL(0) is too weak, LL(k) has a too large table.

- Q4: key differences between top-down and bottom-up parsing?

Top-down is based on leftmost derivation;

bottom-up is the reverse of rightmost derivation.

- Q5: key operations of bottom-up parsing?

Shift: pushes a terminal on the stack.

Reduce: pops RHS and pushes LHS.

$S \rightarrow Aa$
 $A \rightarrow BD$
 $B \rightarrow b$
 $B \rightarrow \epsilon$
 $D \rightarrow d$
 $D \rightarrow \epsilon$

The Example

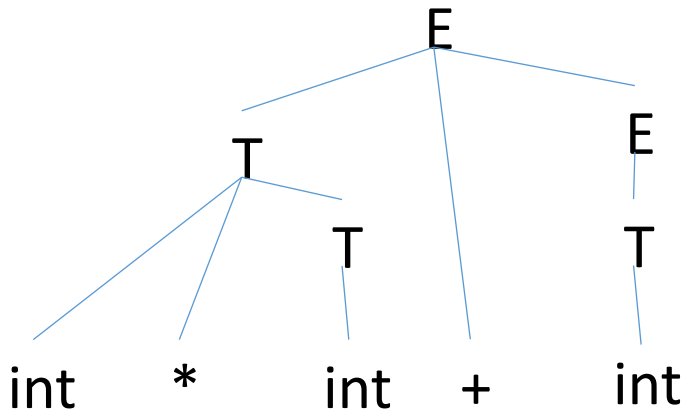
- Grammar

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

- String

int * int + int



Step	Operation
#int * int + int	Shift
int# * int + int	Shift
int * #int + int	Shift
int * int # + int	Reduce $T \rightarrow \text{int}$
int * T # + int	Reduce $T \rightarrow \text{int} * T$
T # + int	Shift
T + # int	Shift
T + int #	Shift
T + T #	Reduce $T \rightarrow \text{int}$
T + E #	Reduce $E \rightarrow T$
E #	Reduce $E \rightarrow T + E$

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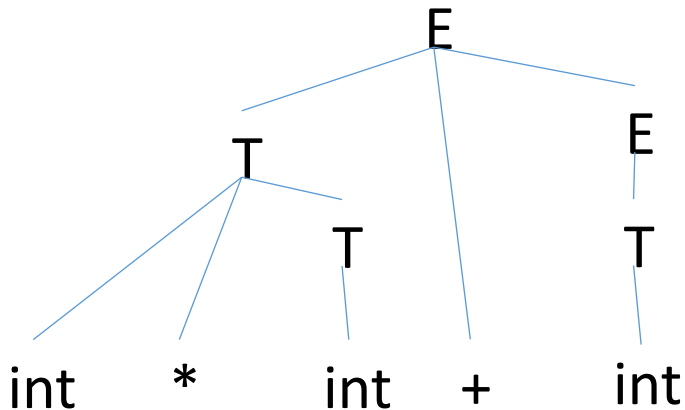
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Handle[句柄]

- A **handle** of a sentential form is a substring α such that:
 - α matches the RHS of a production $A \rightarrow \alpha$; and [匹配规则]
 - replacing α by the LHS A represents a step in the reverse of a rightmost derivation of S [推进解析]
- Definition: let $\alpha\beta\omega$ be a sentential form where:
 - α, β is a string of terminals and non-terminals (yet to be derived)
 - ω is a string of terminals (already derived)
 - Then β is a **handle** of $\alpha\beta\omega$ if: $S \Rightarrow_{rm}^* \alpha X \omega \Rightarrow \alpha\beta\omega$ by a rightmost derivation (apply rule $X \rightarrow \beta$)
- We only want to reduce at handles, and there is exactly one handle per sentential form
 - But where to find it?

Some Concepts[一些概念]

- A **right-sentential form**[最右句型] is a sentential form that occurs in the rightmost derivation of some sentence
- A **phrase**[短语] is a subsequence of a sentential form that is eventually “reduced” to a single non-terminal
 - β is a phrase of the right sentential form γ iff $S \Rightarrow^* \gamma = \alpha_1 A \alpha_2 \Rightarrow^+ \alpha_1 \beta \alpha_2$
 - a string consisting of all of the leaves of the partial parse tree that is rooted at one particular internal node of the whole parse tree[一个句型的语法树中任一子树叶结点所组成的符号串都是该句型的短语]
- A **simple phrase**[直接短语] is a phrase that is reduced in a single step
 - β is a simple phrase of the right sentential form γ iff $S \Rightarrow^* \gamma = \alpha_1 A \alpha_2 \Rightarrow \alpha_1 \beta \alpha_2$
- The **handle** is the leftmost simple phrase[最左直接短语]

Handle: Example

- Grammar

$E \rightarrow T+E|T$

$T \rightarrow \text{int}*T | \text{int} | (E)$

- String

$\text{int} * \text{int} + \text{int}$

Step	Operation
#int * int + int	Shift
int# * int + int	Shift
int * #int + int	Shift
int * int # + int	Reduce $T \rightarrow \text{int}$
int * T # + int	Reduce $T \rightarrow \text{int}*T$
T # + int	Shift
T + # int	Shift
T + int #	Shift
T + T #	Reduce $T \rightarrow \text{int}$
T + E #	Reduce $E \rightarrow T$
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T + # int	Shift
T + int #	Shift
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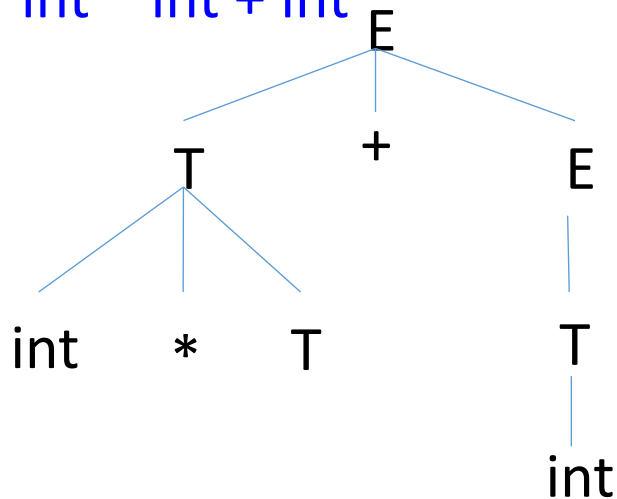
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E #	Reduce $E \rightarrow T + E$

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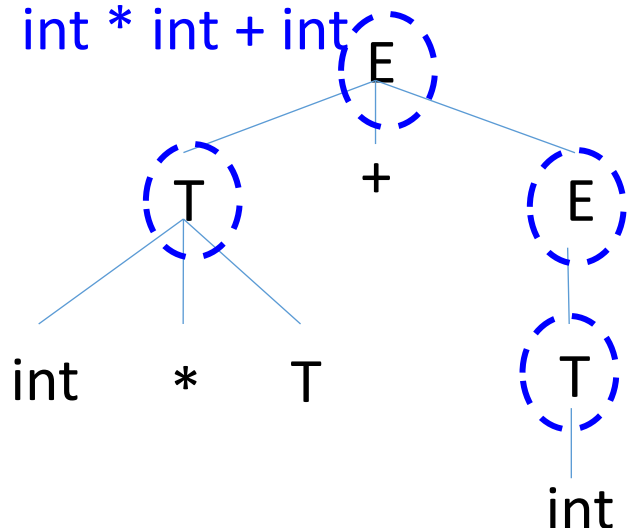
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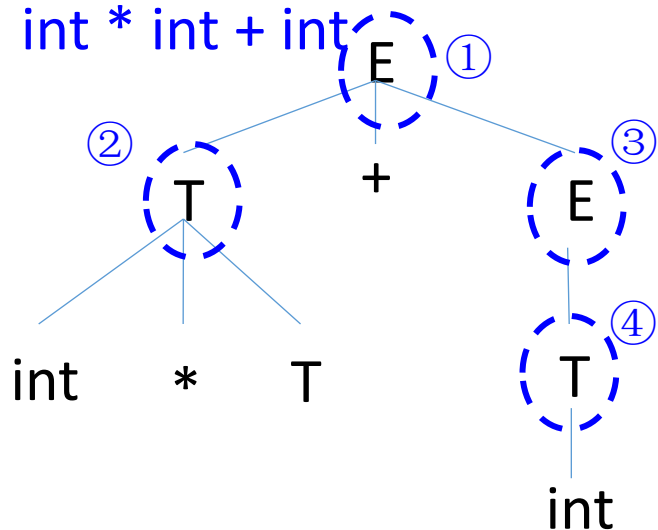
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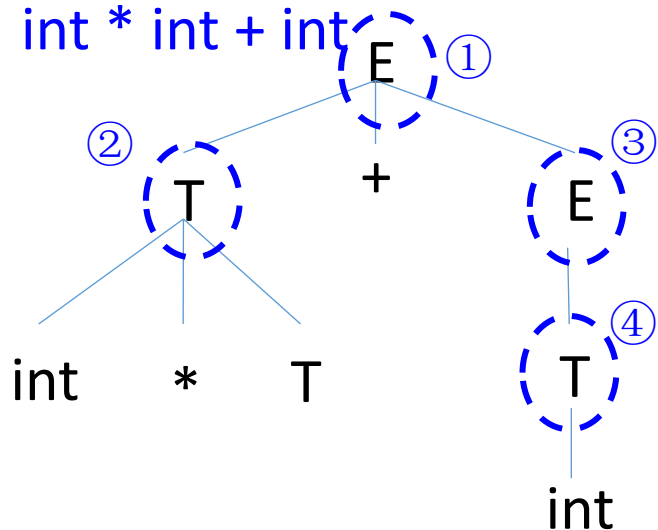
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- String

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Phrase: int * T + int, int * T, int

Simple phrase: int * T, int

Handle: int * T

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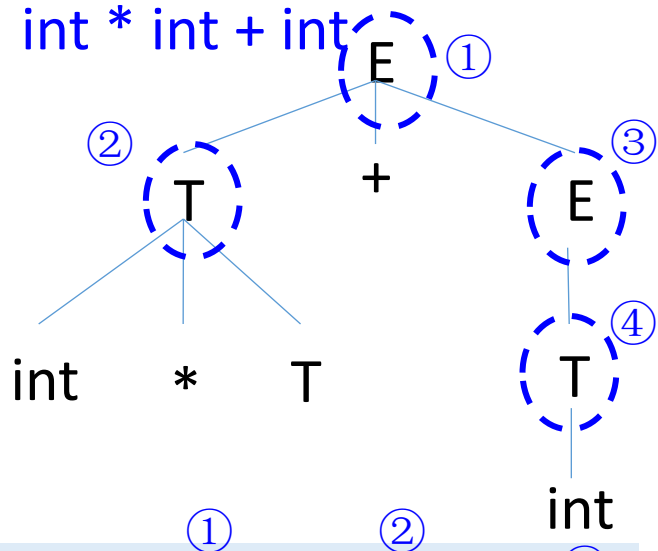
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Phrase: int * T + int, int * T, int^③

Simple phrase: int * T, int^④

Handle: int * T

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One More Example

- Grammar

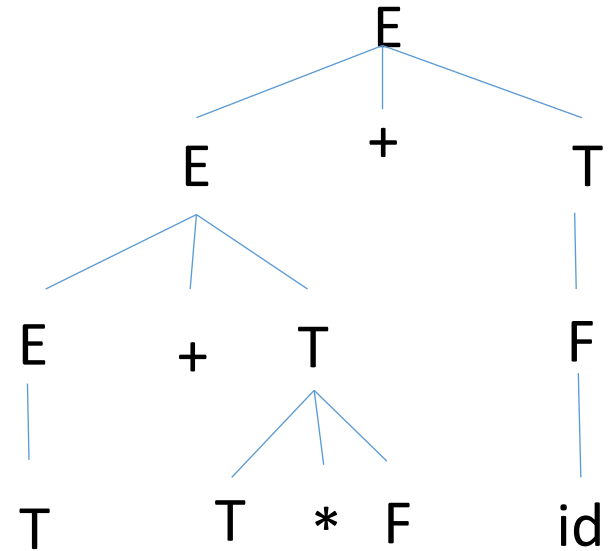
$E \rightarrow E+T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid \text{id}$

- Sentential form:

- Phrase:



One More Example

- Grammar

$E \rightarrow E+T \mid T$

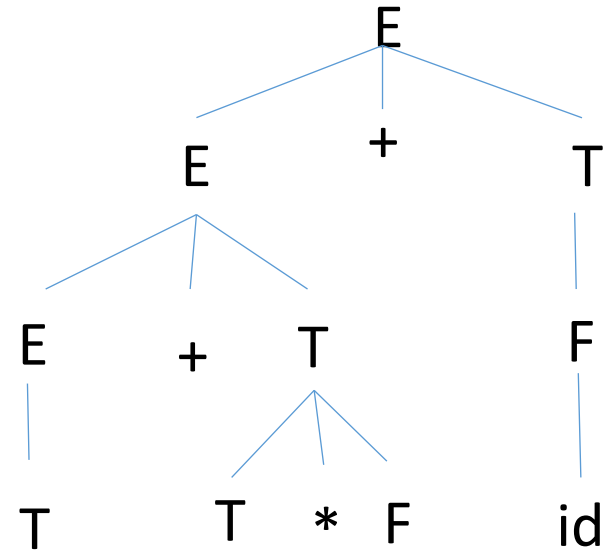
$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid \text{id}$

- Sentential form:

– $T + T * F + \text{id}$

- Phrase:



One More Example

- Grammar

$E \rightarrow E+T \mid T$

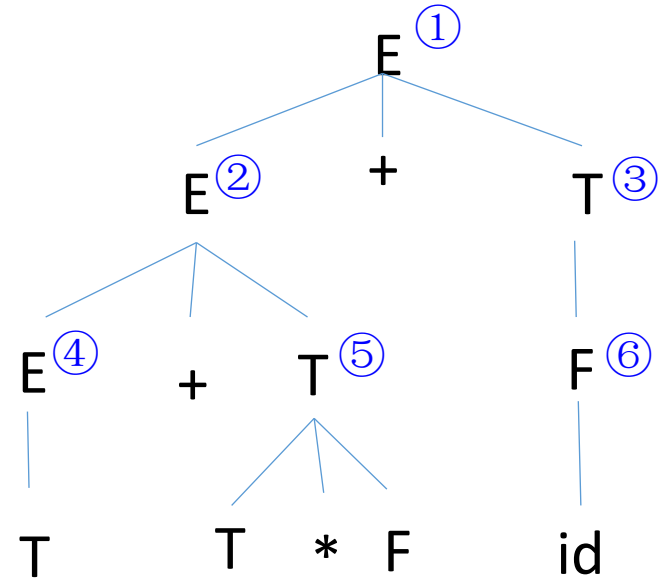
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One More Example

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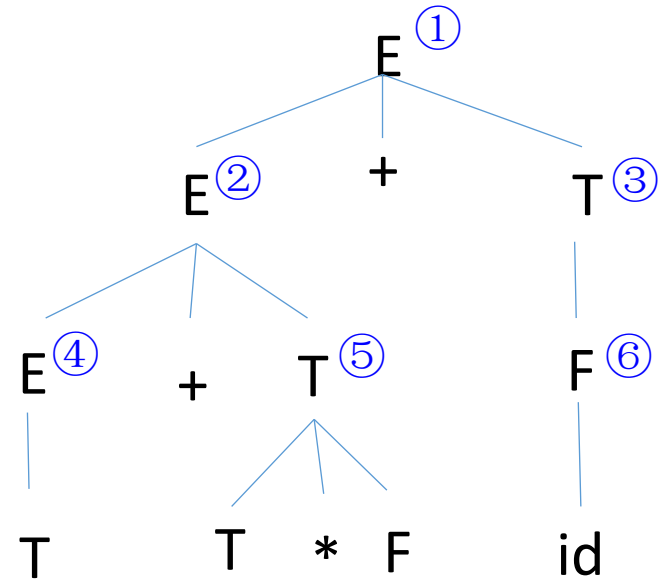
– $T + T * F + \text{id}$ ①

– $T + T * F$ ②

– T ④

– $T * F$ ⑤

– id ③ ⑥



One More Example

- Grammar

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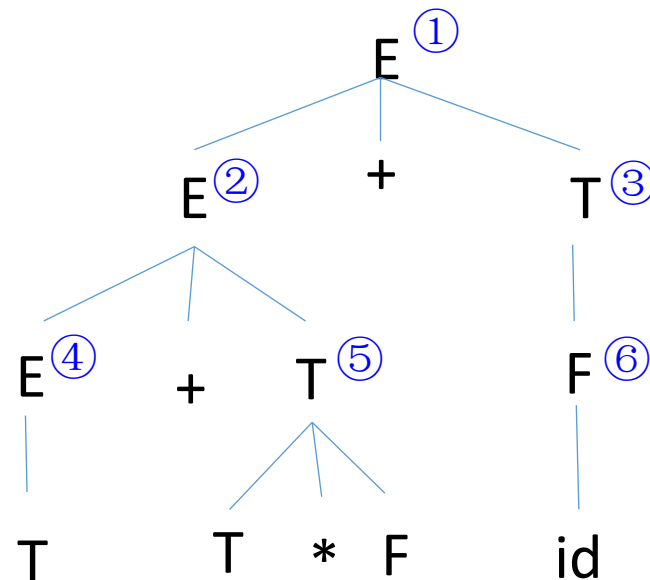
– $T + T * F$ ②

– T ④

– $T * F$ ⑤

– id ③ ⑥

} Simple phrase



One More Example

- Grammar

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$T \rightarrow T * F \mid F$

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- Phrase:

– $T + T * F + \text{id}$ ①

– $T + T * F$ ②

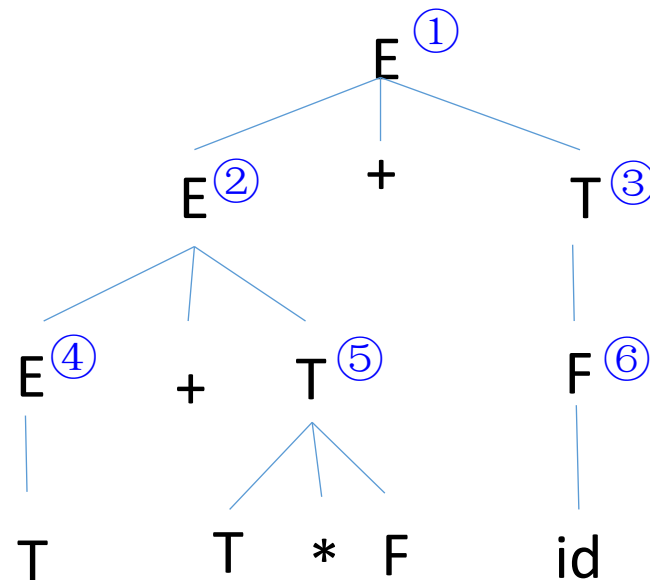
– T ④

– $T * F$ ⑤

– id ③ ⑥

Handle

Simple phrase



Handle Always Occurs at Stack Top

- Why can't a handle occur on right side of #? [不在栈外]

int * #int + int	Shift
int * int # + int	Reduce $T \rightarrow \text{int}$
int * T # + int	Reduce $T \rightarrow \text{int} * T$

Handle Always Occurs at Stack Top

- Why can't a handle occur on right side of #?[不在栈外]
 - It can
 - But handle will eventually be shifted in, placing it at top of stack
 - In $\text{int} * \# \text{int} + \text{int} \Rightarrow \text{int} * \text{int} \# + \text{int}$, int is eventually shifted to the top

$\text{int} * \# \text{int} + \text{int}$	Shift
$\text{int} * \text{int} \# + \text{int}$	Reduce $T \rightarrow \text{int}$
$\text{int} * T \# + \text{int}$	Reduce $T \rightarrow \text{int} * T$

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- Why can't a handle occur on left side of #, i.e., in middle of the stack?[不在栈'中']

$\text{int} * \# \text{int} + \text{int}$	Shift
$\text{int} * \text{int} \# + \text{int}$	Reduce $T \rightarrow \text{int}$
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 - Means parser shifted when it could have reduced when the handle was on top
 - If parser eagerly reduces when handle is at top of stack, never occurs

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 - Can $\text{int} * \text{int} + \# \text{int}$ occur? No.
 - Means parser shifted when it could have reduced when the handle was on top
 - If parser eagerly reduces when handle is at top of stack, never occurs
- Makes life easier for parser (need only access top of stack)

$\text{int} * \# \text{int} + \text{int}$	Shift
$\text{int} * \text{int} \# + \text{int}$	Reduce $T \rightarrow \text{int}$
$\text{int} * T \# + \text{int}$	Reduce $T \rightarrow \text{int} * T$

Viabale Prefix[活前缀]

- In shift-reduce parsing, the stack contents are always a **viable prefix**
 - A prefix of some right-sentential form that ends no further right than the end of the handle of that right-sentential form
 - A viable prefix has a handle at its rightmost end
 - Stack content is always a viable prefix, guaranteeing the shift / reduce is on the right track[活前缀说明移进规约是正确的]
- 定义：一个可行前缀是一个最右句型的前缀，并且它没有越过该最右句型的最右句柄的右端
 - 举例： $S \Rightarrow bBa \Rightarrow bbAa$ ，这里句柄是 bA ，因此可行前缀包括 bA 的所有前缀（包括 b, bb, bbA ），但不能是 $bbAa$ （因为越过了句柄）

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int * #int + int	Shift
int * int # + int	Reduce $T \rightarrow int$
int * T # + int	Reduce $T \rightarrow int*T$
T # + int	Shift

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... $\Rightarrow T + int \Rightarrow int * T + int$

Handle: $int * T$

Viable prefix: $int, int *, int * T$

$int * \#int + int$	Shift
$int * int \# + int$	Reduce $T \rightarrow int$
$int * T \# + int$	Reduce $T \rightarrow int * T$
$T \# + int$	Shift

Ambiguous Grammars[二义文法]

- Conflicts arise with ambiguous grammars
 - Bottom up parsing predicts action w/ lookahead (just like LL)
 - If there are multiple correct actions, parse table will have conflicts
- Example:
 - Consider the ambiguous grammar $E \rightarrow E * E \mid E + E \mid (E) \mid \text{int}$

Sentential form	Actions	Sentential form	Actions
int * int + int	shift	int * int + int	shift
...
E * E # + int	reduce E → E * E	E * E # + int	shift
E # + int	shift	E * E + # int	shift
E + # int	shift	E * E + int #	reduce E → int
E + int #	reduce E → int	E * E + E #	reduce E → E + E
E + E #	reduce E → E + E	E * E #	reduce E → E * E
E #		E #	

Ambiguous Grammars (cont.)

- In the red step shown, can either shift or reduce by $E \rightarrow E * E$
 - Both okay since precedence of $+$ and $*$ not specified in grammar
 - Same problem with associativity of $+$ and $*$
- As usual, remove conflicts due to ambiguity ...
 - 1. Rewrite grammar/parser to encode precedence and associativity
 - Rewriting grammar results in more convoluted grammars
 - Parser tools have other means to encode precedence and association
 - 2. Get rid of remaining ambiguity (e.g. if-then-else)
 - No choice but to modify grammar
- Is ambiguity the only source of conflicts?
 - Limitations in lookahead-based prediction can cause conflicts
 - But these cases are very rare

Properties of Bottom-up Parsing[属性]

- Handles always appear at the **top of the stack**
 - Never in middle of stack
 - Justifies use of stack in shift – reduce parsing
- Results in an easily generalized **shift – reduce** strategy
 - If there is no handle at the top of the stack, shift
 - If there is a handle, reduce to the non-terminal
 - Easy to automate the synthesis of the parser using a table
- Can have conflicts
 - If it is legal to either shift or reduce then there is a shift-reduce conflict
 - If there are two legal reductions, then there is a reduce-reduce conflict
 - Most often occur because of ambiguous grammars
 - In rare cases, because of non-ambiguous grammars not amenable to parser

Types of Bottom-Up Parsers[类型]

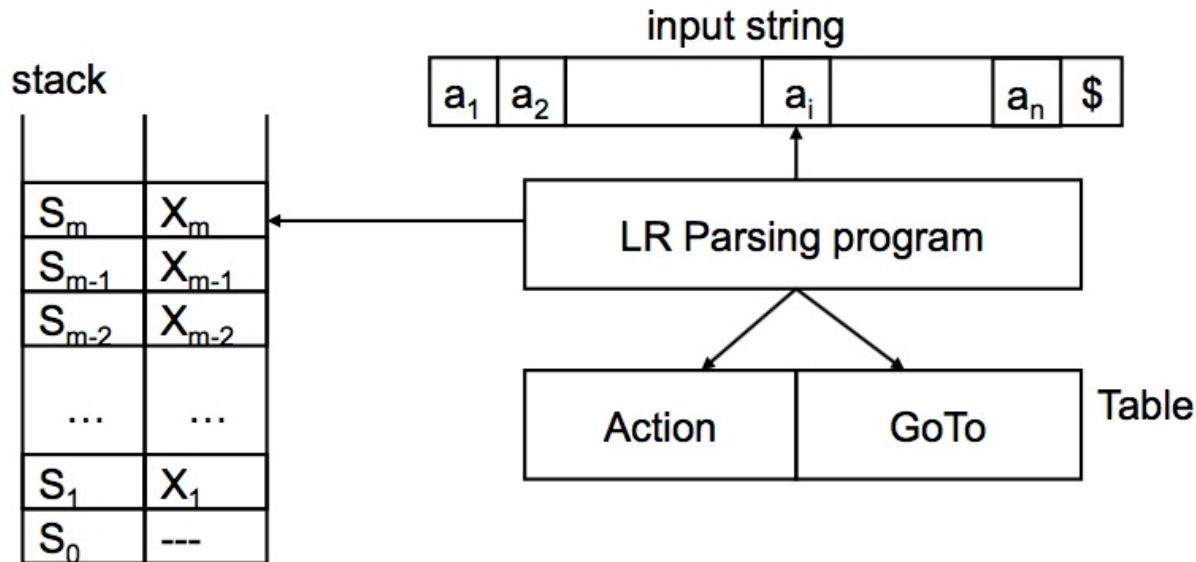
- Types of bottom up parsers
 - Simple precedence parsers[简单优先解析器]
 - Operator precedence parsers[运算符优先解析器]
 - Recursive ascent parsers[递归提升解析器]
 - LR family parsers[LR类解析器]
 - ...

- In this course, we will only discuss **LR family parsers**
 - Efficient, table-driven shift-reduce parsers
 - Most automated tools for bottom-up parsing generate LR family
 - Categories: LR(0), LR(1), SLR, LALR, ...

LR(k) Parser

- **LR(k)**: member of LR family of parsers
 - **L**: scan input from left to right
 - **R**: construct a rightmost derivation in reverse
 - **k**: number of input symbols of lookahead to make decisions
 - $k = 0$ or 1 are of particular interests, is assumed to be 1 when omitted
- Comparison with LL(k) parser[对比]
 - Efficient as LL(k)
 - Linear in time and space to length of input (same as LL(k))
 - Convenient as LL(k)
 - Can generate automatically from grammar – YACC, Bison
 - More complex than LL(k)
 - Harder to debug parser when grammar causes conflicting predictions
 - More powerful than LL(k)
 - Handles more grammars: no left recursion removal, left factoring unneeded
 - Handles more (and most practical) languages: $LL(1) \subset LR(1)$

LR Parser



- The stack holds a sequence of states, $S_0S_1\dots S_m$ (s_m is the top)
 - States are to track where we are in a parse
 - Each grammar symbol X_i is associated with a state s_i
- Contents of stack + input ($X_1X_2\dots X_m a_i\dots a_n$) is a right sentential form
 - If the input string is a member of the language
- Uses $[S_m, a_i]$ to index into parsing table to determine action

Parse Table[分析表]

- LR parsers use two tables: **action table** and **goto table**
 - The two tables are usually combined
 - Action table specifies entries for terminals
 - Goto table specifies entries for non-terminals
- Action table[动作表]
 - $Action[s, a]$ tells the parser what to do when the state on top of the stack is s and terminal a is the next input token
 - Possible actions: **shift, reduce, accept, error**
- Goto table[跳转表]
 - $Goto[s, X]$ indicates the new state to place on top of the stack after a reduction of the non-terminal X while state s is on top of the stack

Possible Actions[可能动作]

- **Shift**

- Transfer the next input symbol onto the top of the stack

- **Reduce**

- If there's a rule $A \rightarrow w$, and if the contents of stack are qw for some q (q may be empty), then we can reduce the stack to qA

- **Accept**

- The special case of reduce: reducing the entire contents of stack to the start symbol with no remaining input
- Last step in a successful parse: have recognized input as a valid sentence

- **Error**

- Cannot reduce, and shifting would create a sequence on the stack that cannot eventually be reduced to the start symbol

Possible Actions (cont.)

- Grammar

$$S \rightarrow E$$

$$E \rightarrow T \mid E + T$$

$$T \rightarrow \text{id} \mid (E)$$

- Input: (id + id)

$$\begin{aligned} - \#(\text{id} + \text{id})\$ \Rightarrow (\text{id}\#\text{id})\$ \Rightarrow (T\#\text{id})\$ \Rightarrow (E\#\text{id})\$ \Rightarrow (E+\text{id}\#)\$ \Rightarrow \\ (E+T\#)\$ \Rightarrow (E\#)\$ \Rightarrow (E)\#\$ \Rightarrow T\#\$ \Rightarrow E\#\$ \Rightarrow S\#\$ \end{aligned}$$

- Input: id+)

$$- \#\text{id}+)\$ \Rightarrow \text{id}\#\text{)}\$ \Rightarrow T\#\text{)}\$ \Rightarrow E\#\text{)}\$ \Rightarrow E+\text{)}\$ \dots$$

Example: Parse Table

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

- Table entry:

- si : shifts the input symbol and moves to state i (i.e., push state on stack)
- rj : reduce by production numbered j
- acc: accept
- blank: error

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state \rightarrow 0

b a b

symbol \rightarrow \$

b a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state \rightarrow 0

b a b

symbol \rightarrow \$

b a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state \rightarrow 0

b a b

symbol \rightarrow \$

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state \rightarrow 0

b a b

symbol \rightarrow \$ b

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state \rightarrow 0 4
 symbol \rightarrow \$ b a b \$
 b a b

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state → 0 4
symbol → \$ b
a b \$

b a b

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state \rightarrow 0 4

b a b

symbol \rightarrow \$ b

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state \rightarrow 0 4

b a b

symbol \rightarrow \$ b

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state \rightarrow 0 4

b a b

symbol \rightarrow \$

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state \rightarrow 0

b a b

symbol \rightarrow \$

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state \rightarrow 0

b a b

symbol \rightarrow \$ B

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0

symbol \rightarrow \$ B

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0

symbol \rightarrow \$ B

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0

symbol \rightarrow \$ B

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2

symbol \rightarrow \$ B

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2

symbol \rightarrow \$ B

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2

symbol \rightarrow \$ B

a b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2

symbol \rightarrow \$ B

b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2

symbol \rightarrow \$ B a

b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2 3

symbol \rightarrow \$ B a

b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2 3

symbol \rightarrow \$ B a

b \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2 3

symbol \rightarrow \$ B a

\$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2 3

symbol \rightarrow \$ B a b

\$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2 3 4

symbol \rightarrow \$ B a b

\$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2 3 4

symbol \rightarrow \$ B a b

\$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2 3 4

symbol \rightarrow \$ B a b

\$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2 3

symbol \rightarrow \$ B a

\$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state \rightarrow 0 2 3

symbol \rightarrow \$ B a B

\$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B B
 | |
 b a b

state \rightarrow 0 2 3
 symbol \rightarrow \$ B a B \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B B
 | |
 b a b

state \rightarrow 0 2 3
 symbol \rightarrow \$ B a B \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B B
 $|$ $|$
 b a b

state \rightarrow 0 2 3
 symbol \rightarrow \$ B a B \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B B
 | |
 b a b

state \rightarrow 0 2 3 6
 symbol \rightarrow \$ B a B \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B B
 | |
 b a b

state \rightarrow 0 2 3 6
 symbol \rightarrow \$ B a B \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B B
 | |
 b a b

state \rightarrow 0 2 3 6
 symbol \rightarrow \$ B a B \$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B B
 | |
 b a b

state \rightarrow 0 2
 symbol \rightarrow \$ B

\$

Example: Parse Table (cont.)

Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B B
 | |
 b a b

state \rightarrow 0 2
 symbol \rightarrow \$ B B \$

Example: Parse Table (cont.)

Grammar:

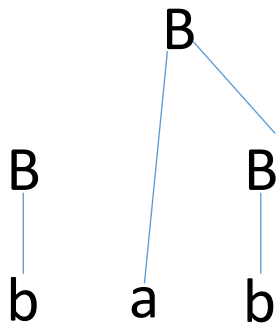
(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		



state \rightarrow 0 2

symbol \rightarrow \$ B B

\$

Example: Parse Table (cont.)

Grammar:

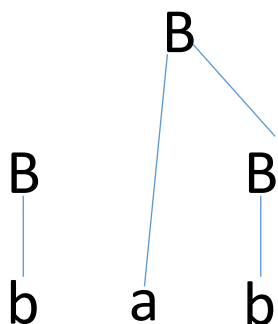
(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		



state \rightarrow 0 2
 symbol \rightarrow \$ B B \$

Example: Parse Table (cont.)

Grammar:

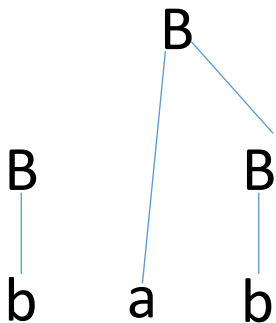
(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		



state \rightarrow 0 2

symbol \rightarrow \$ B B

\$

Example: Parse Table (cont.)

Grammar:

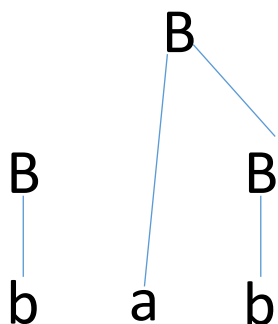
(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		



state \rightarrow 0 2 5
 symbol \rightarrow \$ B B \$

Example: Parse Table (cont.)

Grammar:

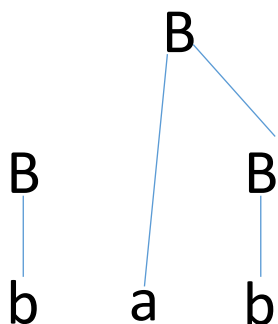
(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		



state \rightarrow 0 2 5
 symbol \rightarrow \$ B B \$

Example: Parse Table (cont.)

Grammar:

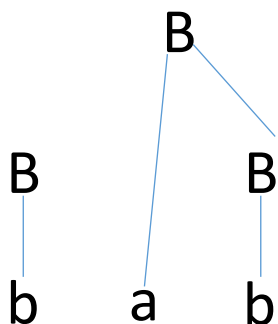
(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		



state \rightarrow 0 2 5
 symbol \rightarrow \$ B B \$

Example: Parse Table (cont.)

Grammar:

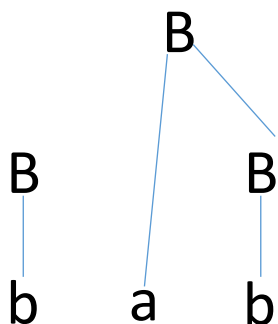
(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		



state \rightarrow 0

symbol \rightarrow \$

\$

Example: Parse Table (cont.)

Grammar:

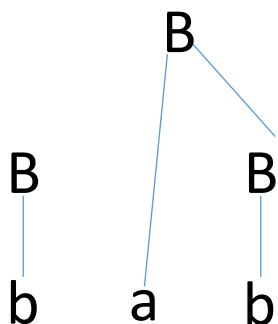
(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		



state \rightarrow 0
 symbol \rightarrow \$ S

\$

Example: Parse Table (cont.)

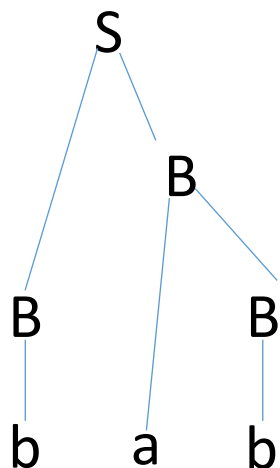
Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab



state \rightarrow 0
 symbol \rightarrow \$ S

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

Example: Parse Table (cont.)

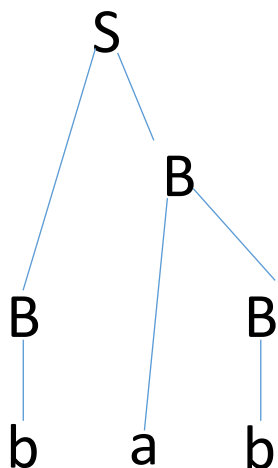
Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab



state \rightarrow 0
symbol \rightarrow \$ S

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

Example: Parse Table (cont.)

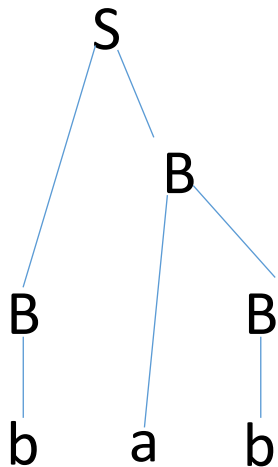
Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab



state \rightarrow 0
 symbol \rightarrow \$ S

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

Example: Parse Table (cont.)

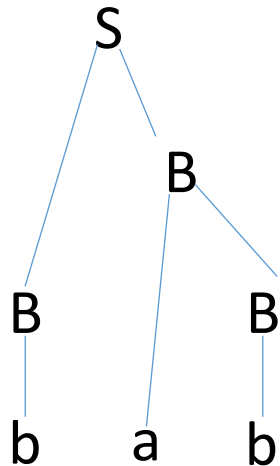
Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab



state \rightarrow 0 1
 symbol \rightarrow \$ S

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

Example: Parse Table (cont.)

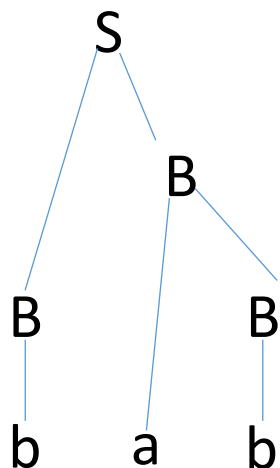
Grammar:

(1) $S \rightarrow BB$

(2) $B \rightarrow aB$

(3) $B \rightarrow b$

String: bab



state \rightarrow 0 1
 symbol \rightarrow \$ S

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		